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# NBS TECHNICAL NOTE 572

## On Computer Performance Measurement Programming

### Measuring Indexing Adroitness By Isolating Complex Primes

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By Isolating Complex Primes**

George W. Reitwiesner

Office of Information Processing Standards  
Center for Computer Sciences and Technology  
National Bureau of Standards  
Washington, D.C. 20234



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On Computer Performance Measurement Programming  
Measuring Indexing Adroitness by Isolating Complex Primes

George W. Reitwiesner

This writing, describing a computer performance test program, is concerned not primarily with specific measurements, but rather with a procedure for making measurement regarding specific properties of computer operation.

The program is written in a particular problem-oriented programming language; therefore assessment perforce spans the effects of the computer hardware, of the programming language, and of the intervening compiler processes.

The objective of the test is to assess adroitness in certain indexing operations. Assessment is accomplished by measuring execution time of a recursive programming loop.

The test problem was chosen as a convenient artifice to use certain specific indexing-type operations in the programming employed for solution.

The test program performs a simple computation for which the solution is completely definitive, yet for which both the solution and the time for achieving it are variable under parameters whose values are introduced as program input data.

Key words: Assessment; complex; composite; computer; criteria; evaluation; Gaussian primes; indexing; measurement; performance; prime; program; test.

## Introduction

This paper describes a digital computer performance test program which was developed to meet a particular set of test criteria.

A programmed digital computer performance test should assess some single basic characteristic of digital computer performance. It should do so objectively in the sense of being insulated against possible effects of especial hardware and logical design features. It should be elevated above the mundane by a touch of the esoteric, yet its employment of the characteristic under test should occur in as natural an environment as possible. It should provide a unique result under conditions which permit the recording of some measurement (such as program execution time) to assess performance in obtaining that result. That result should not be reasonably determinable by any other means, should be variable under the control of parameters whose values are entered as input data at the outset of the performance of the test, and should be precisely reproducible on all processors which operate under the same programming language.

The test program described in this writing has been designed to permit objective assessment of the single characteristic of adroitness in indexing operations, as revealed in the execution of a particularly constructed innermost recursive programming loop. It is written in the FORTRAN problem-oriented language. Two principal indices are used in this innermost loop; one index is administered under a DO statement, and the other is advanced by an integer arithmetic statement; both indices are used in integer arithmetic comparison (IF) statements, and both are used to select a particular element in a two-dimensional array.

When it runs to normal completion, the test program performs the moderately esoteric exercise of isolating all the complex primes over a selected region of the complex plane. In effecting this isolation, the innermost loop employs indexing operations of the type described above in the rather natural programming situation of traversing along the outer edge of an origin-centered circular arc: to locate points at which that arc coincides with intersections of integer grid lines.

Assessment is made by measuring program execution time; and the data which are printed upon completion of the test run include a plot of the unique pattern of complex primes over the selected region, together with certain peculiar counts, such as the multiplicity of recognition of compositeness among non-primes and the frequency of a specially selected index change during execution.

Parametric control is available for the assignment of the exact dimensions of the selected region and, optionally, to effect premature program termination upon completion of a preassigned portion of the total work required for completing the isolation. Apart from program execution time, the results are precisely reproducible on any processor operating under the programming language used.



## General Discussion

This writing describes a test program for a computer. The first half of the text describes the test in general terms; the second half presents an analysis of pertinent details.

The objective of the test is to assess adroitness in indexing operations. Indexing in computer programming is a complicated subject; the test described here covers only one aspect of that subject. It assesses the adroitness with which two indices perform conventional operations under non-conventional control and non-conventional operations under conventional control: both indices are used to access elements in a two-dimensional array, and both are used in arithmetic control statements; one index is administered under a loop-control statement, and the other is advanced by an arithmetic assignment statement.

The program is written in FORTRAN. It is sufficiently simple and brief that its translation to another programming language may be performed easily.

The assessment, perforce, spans the effects of the computer hardware and logical design, of the computer-instruction-level language which is generated by the compiler, and of the technique by which the compiler effects the indexing operations required.

Assessment is made by measuring the time of execution of a multi-echelon recursive programming loop. Two time-measuring techniques are provided: interrogating a program-accessible internal clock, if available, and manual timing between pauses. Either, or both, may be selected.

The test program has not been designed to represent an efficient computational procedure. We are not concerned here with efficiency in that sense. We are concerned with indexing adroitness.

Adroitness in basic aspects of indexing operations is demanded by the particular manner in which the innermost recursive loop is programmed. The pertinent features of the structure of this loop are shown in the sample program of Table I.

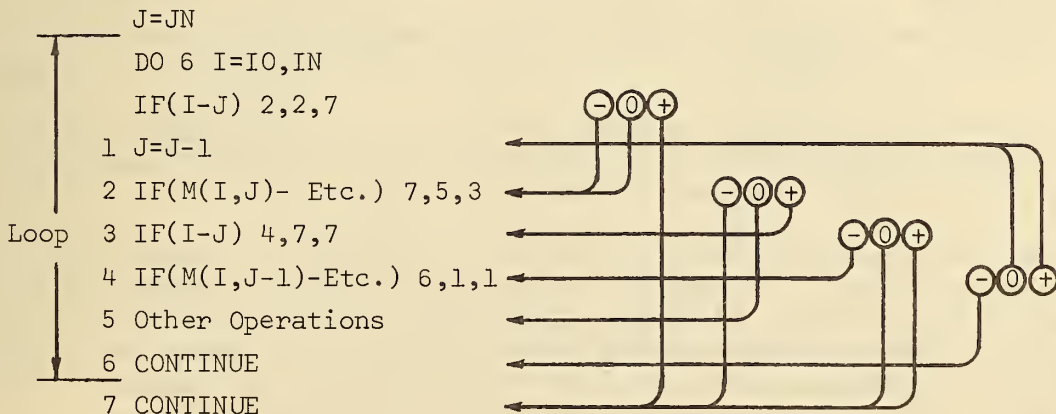


Table I. Sample Program

This sample program, being illustrative, is not complete; and it is detailwise slightly different from that employed in the test program; however, insofar as concerns the pertinent features to which we address our argument, it is logically essentially equivalent to the innermost loop of that program.

A feature of this loop which is of especial interest in regard to demanding indexing adroitness is that each of its first six statements, beginning with the DO 6 I=I0,IN statement and ending with the statement numbered 4, is a basic block terminating statement as defined in paragraph 10.2.7 of the descriptions of the FORTRAN language both in American National Standards Institute document X3.9-1966 and in the Communications of the Association for Computing Machinery, vol. 7, No. 10, October 1964, pages 623-624.

Our interest lies in the net performance resulting from the techniques by which indices span their ranges of application and the manners in which they are employed in the recursive looping which they govern.

In our sample program the indices span their ranges under two techniques: one index is governed by the conventional DO 6 I=I0,IN statement, and the other is (negatively) advanced by the J=J-1 conventional integer arithmetic statement. In the body of the recursion, each index is employed in each of two ways: to access a two-dimensional array (M(I,J)), and in effecting integer magnitude comparisons (IF(I-J) Etc.). Other operations occur in the innermost loop, but (for representative values of the governing parameters) sufficiently infrequently to be disregarded.

The test problem employs only integer numerical data.

The test problem performs a simple computation for which the solution is completely definitive, yet for which both the solution and the time required for achieving it are variable under control of three integer parameters. The roles of these three parameters are described in the following several paragraphs.

The test program employs complex integers, also known as Gaussean integers: complex numbers with integer coefficients. It searches a prescribed region of the complex plane to locate all composite complex integers: those which are the products of other complex integers of non-unit modulus.

Two of the parameters limit the range of interest over the complex plane to an origin-centered cross, with four axes of symmetry which separate the region into eight congruent sectors, as shown in Figure 1.

The larger of these parameters is the upper limit which the magnitude of either coefficient of a complex number is permitted to assume; thus it bounds the coefficient of larger magnitude. The other of these parameters is the upper limit which the magnitude of the other coefficient is permitted to assume; thus it bounds the coefficient of smaller magnitude. (When these parameters are equal, the cross in Figure 1 degenerates to a square.)

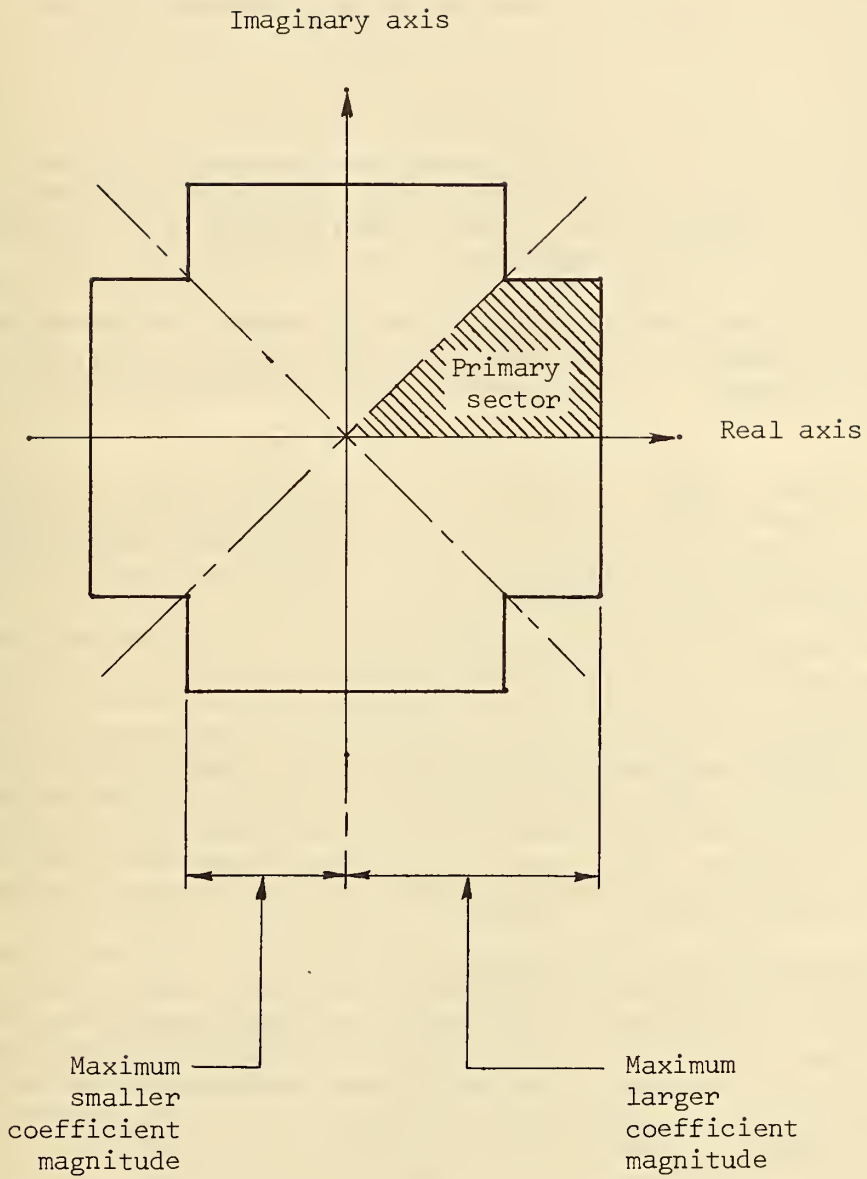


Figure 1.

The test program can and does employ the eight-fold symmetry remarked above. It develops data which, through that symmetry, apply throughout the region of interest. It does so, using only complex integers which are contained in (inside or on a boundary of) a particular one of the eight sectors: that one for which both coefficients are non-negative and for which the imaginary coefficient does not exceed the real coefficient. We call this the primary sector; it is shaded in Figure 1.

We concentrate attention on the primary sector.

Complex integers on the real-equals-imaginary diagonal boundary of this sector we call diagonal complex integers. They have the form:  $x+xi$ . Except for  $x=1$ , they all are composite (factoring into  $x$  and  $1+i$ ); and, trivially,  $1+i$  is not composite.

The test program requires the performance of comparisons among moduli of complex integers. These comparisons are made, instead, upon their corresponding squares. This insures that only integer numerical data are employed.

For each complex integer contained in the primary sector, the program records two items of data: the square of the modulus, and a counter-marker which counts the number of times, if any, that that complex integer is recognized as composite.

There is an exception: only one data item is recorded for each diagonal complex integer: the square of the modulus.

Taking advantage of the eight-fold symmetry, the program stores the two (triangularly-arrayed) sets of data in a single square array: the squares of the moduli are stored on and below the principal diagonal of that array; and the counter-markers are stored above that diagonal. The program maintains a separate single count of the total number of times compositeness is recognized among all the diagonal complex integers.

The third parameter controls program termination: it is the upper limit on the number of times the program will recognize compositeness among diagonal complex integers. The program terminates prematurely when that number reaches the value of this parameter; however, the program runs to a normal termination when the value of this parameter is zero or exceeds the maximum that that number can reach under the conditions (sector dimensions) imposed by the other parameters.

When it runs to normal termination, the test program searches out and labels (with a count) every composite complex integer of the primary sector, leaving unlabeled (uncounted) only the complex primes (also called Gaussian primes), contained in the region of interest. This distinction (composite vs. prime) is not necessarily valid when the program terminates prematurely under the control of the third parameter.



The precise search procedure is detailed below. In general terms it is a counterpart (albeit a redundant and cumbersome one) to the familiar sieve procedure for isolating real primes in a sequence of real numbers by eliminating all multiples of smaller integers (primes) other than 1.

We are not principally interested in, per se, identifying complex primes.

We employ our sieve search procedure for isolating complex primes merely as an (academically appealing) artifice to afford many-fold reiteration of a particular inner programming loop which has properties of interest to us, and to yield a totally definitive result. For any particular selection of values of the three governing parameters, this definitive result embodies: the (conditionally valid) distinction of composite vs. prime; the compositeness recognition counts remarked above; and one further count which is incorporated for incidental information: the program counts the number of times the  $J=J-1$  index change represented by statement 1 of the sample program of Table 1 is performed throughout the total course of execution.

Upon termination, the program prints two sets of data.

One set contains a miscellany:

- (1) the three governing parameters,
- (2) the compositeness recognition count for diagonal complex integers,
- (3) the sum of all compositeness recognition counts for non-diagonal complex integers,
- (4) the sum of the above two counts (3) and (2),
- (5) the largest compositeness recognition count among the non-diagonal complex integers and the coefficients of the complex integer associated thereto (or of a particular one of them if duplicate maximum counts exist),
- (6) the number of executions performed of the  $J=J-1$  index change of statement 1, and
- (7) the execution time of the solution, in seconds. (The execution time is set to zero when internal timing is not employed.)

The other set is a two-dimensional plot of the complex integers which were not recognized as composite, adjusted to include  $1+i$ .

The test program is displayed in appendix A.

It employs exactly one input data card, typified in one of the early comment lines. The first five columns of this card contain certain data which are used for the plot. The next 25 columns contain five five-digit parameters: the first three of these parameters have been accounted above; the last two are auxiliary parameters which select timing and printing options as described in further comment lines. (When internal timing is employed, a suitable factor must be defined (where indicated by the appropriate comment line in the programming) to convert the measured internal timing to seconds.)

A sample output of the test program is displayed in Appendix B.



## Analysis of Details

The remainder of this writing presents a more precise analysis of the problem solved by the test program, and of the particular programming procedure employed. It does this in three steps, covering: general matters, complex integer multiplication, and programming details.

For analytical precision, we enumerate the eight equal sectors of the complex plane as I, II, ... VIII, proceeding counterclockwise, beginning with the primary sector which is shaded in Figure 1.

We denote the generic complex integer  $f$ , of integer coefficients  $g$  and  $h$ , as  $f=gr+hi$ , where  $r$  and  $i$  are unit real and imaginary vectors.

We denote the square of the modulus as  $\text{modsq}$  or  $m()$ ; thus the  $\text{modsq}$  of  $f=gr+hi$  is  $m(f) = g^2 + h^2$ .

We have defined (in the text above) the three terms: complex integer, composite complex integer, and complex prime.

We recognize four complex integers of unit modulus:  $+r$ ,  $-r$ ,  $+i$ ,  $-i$ ; we call them unit complex integers or unit complex primes.

For non-negative  $g$  and  $h$ , there are in general exactly eight composite complex integers whose coefficients (magnitudes) are  $g$  and  $h$  (in either order): we refer to them as derivants of each other and denote them  $f(k,j)$ , as follows:

$$\begin{array}{ll} f(0,0) = gr+hi & f(1,0) = gr-hi \\ f(0,1) = -hr+gi & f(1,1) = -hr-gi \\ f(0,2) = -gr-hi & f(1,2) = -gr+hi \\ f(0,3) = hr-gi & f(1,3) = hr+gi. \end{array} \quad (1)$$

They derive from each other by the rules:

$$\begin{array}{l} f(k,j+1) = (-1)^k(i)(f(k,j)) \\ f(k+1,j) = \text{conjugate of } f(k,j), \end{array} \quad (2)$$

where  $k+1$  is taken modulo 2, and  $j+1$  is taken modulo 4; i.e.

$$\begin{array}{ll} f(2,j) = f(0,j) & (\text{independent of } j) \\ f(k,4) = f(k,0) & (\text{independent of } k). \end{array} \quad (3)$$

Through obvious symmetries, one derivant of any complex integer is contained in each of the eight sectors of the complex plane, and all eight have equal  $\text{modsq}$ :  $m(f(k,j)) = g^2 + h^2$ , independent of  $k$  and  $j$ .

(We do not belabor the degenerate cases in which the eight derivants of a complex integer are not all distinct, viz: when they coincide in pairs because their coefficients vanish or have equal magnitude.)

The particular derivant in the primary sector we call the primary derivant; it is  $f(0,0) = gr+hi$ , for non-negative  $g$  and  $h$ , with  $h$  less than (or =)  $g$ .

Figure 2 displays the eight derivants of four complex integers,  $a,b,c,d$ , where (to accommodate analysis of complex integer multiplication) the several derivants of  $c$  and  $d$  represent the products of the derivants of  $a$  and  $b$ , whence  $m(c)=m(d)$  applies, and both  $c$  and  $d$  lie on the arc (shown) of radius equal to the square root of  $m(c)$  and  $m(d)$ .

In Figure 2 we locate the derivants of  $a,b,c,d$  by the symbol  $o$ .

One or more other complex integers may have the same modsq as  $c$  and  $d$  (and thus also lie on the arc in Figure 2) but not be products of derivants of  $a$  and  $b$ ; for later reference, we denote any such complex integer as  $e$  and employ the symbol  $x$  to locate its eight derivants in Figure 2.

We denote the coefficients of  $a,b,c,d$ , and  $e$  according to:

$$\begin{aligned} a(0,0) &= pr+qi & (4) \\ b(0,0) &= sr+ti \\ c(0,0) &= vr+wi \\ d(0,0) &= yr+zi \\ e(0,0) &= mr+ni \quad (\text{if } e \text{ exists}). \end{aligned}$$

Turning now to multiplication, we observe that for arbitrary  $k',j',k'',j''$ , there exist  $8^2 = 64$  ways of multiplying some derivant of  $a(k',j')$  by some derivant of  $b(k'',j'')$ . (Trivially, we do not distinguish the order of multiplication:  $(a)(b)$  is the same as  $(b)(a)$ .) By virtue of the factor  $i$  in (2), there exists a four-fold duplication among the 64 products. Upon elimination of duplications, there remain 16 distinct complex integers in two sets ( $c$  and  $d$ ) of eight derivants (except when coincidence occurs through degeneration).

We aim to accommodate complex integer multiplication in general, while restricting our attention to the primary sector. To do so, we seek a rule for expressing the primary derivant of each of these two sets of products in terms of the primary derivants of the factors.

One member of one set is the product of the primary derivants of the factors; it is, perforce, contained in one of the two sectors I,II; transformation from sector II to sector I under (2) is straightforward. One member of the other set is the product of the primary derivant of one of the factors and the conjugate of the primary derivant of the other factor; it is, perforce, contained in one of the sectors I,VIII; transformation from sector VIII to sector I under (2) is straightforward.

Thus, employing (4), we have the two multiplications

$$\begin{aligned} (a(0,0))(b(0,0)) &= (ps-qt)r + (qs+pt)i = \text{either } c(0,0) \text{ or } c(1,3) & (5) \\ (a(0,0))(b(1,0)) &= (ps+qt)r + (qs-pt)i = \text{either } d(0,0) \text{ or } d(1,0), \end{aligned}$$

and the rule we seek is

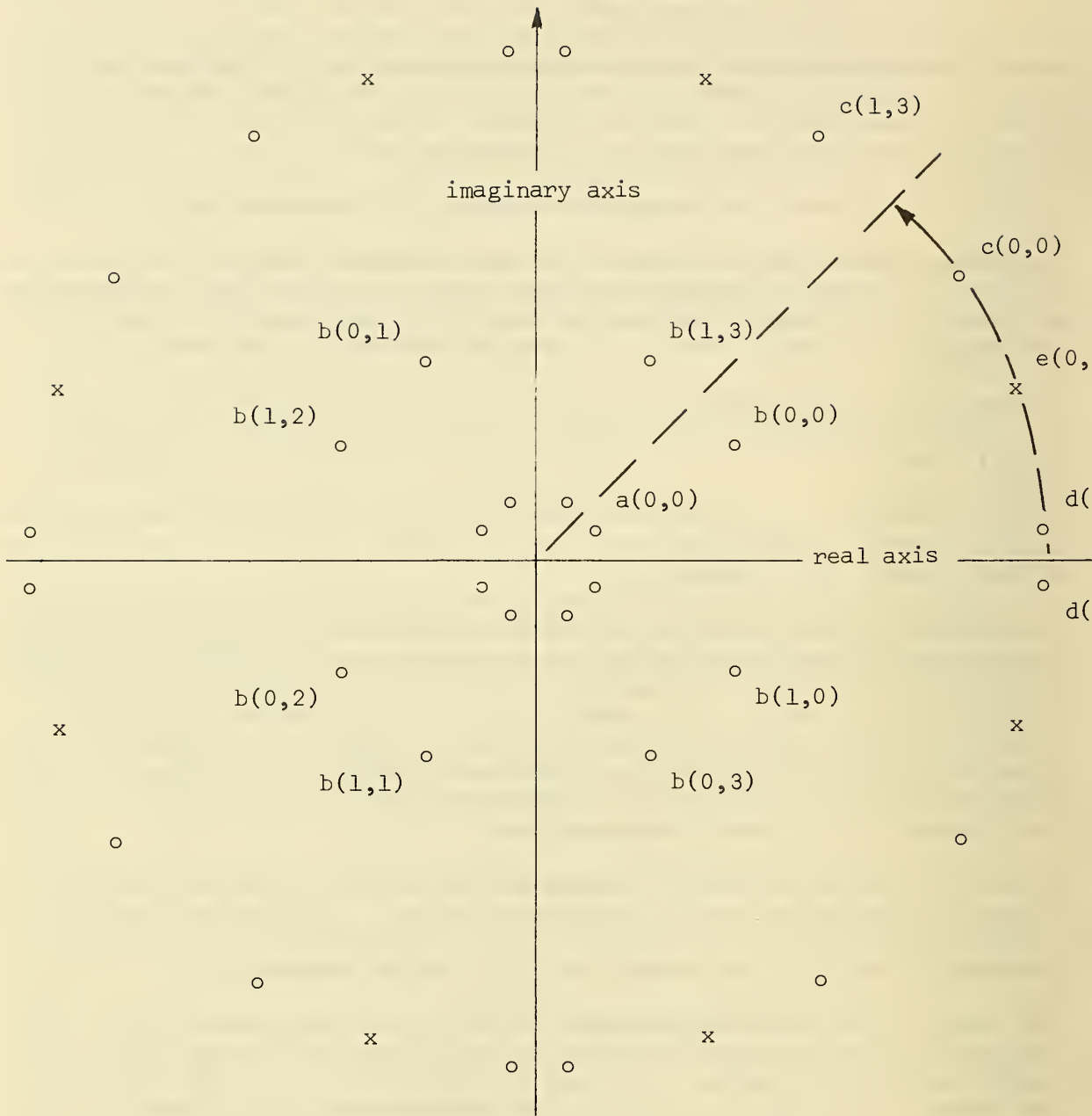


Figure 2.

$$\begin{aligned} c(0,0) &= (\max((ps-qt),(pt+qs)))r + (\min((ps-qt),(pt+rs)))i & (6) \\ d(0,0) &= (ps+qt)r + (\text{magnitude of } (pt-qs))i. \end{aligned}$$

In specific illustration, we consider the assignments

$$\begin{array}{llll} p=2 & q=1 & \text{whence } a(0,0) = 2r + i & \text{and } m(a) = 5 \\ s=7 & t=4 & \text{whence } b(0,0) = 7r + 4i & \text{and } m(b) = 65 \\ v=15 & w=10 & \text{whence } c(0,0) = 15r + 10i & \text{and } m(c) = 325 \\ y=18 & z=1 & \text{whence } d(0,0) = 18r + i & \text{and } m(d) = 325. \end{array} \quad (7)$$

Then, clearly, for

$$\begin{aligned} a(0,0) &= 2r + i & (7') \\ b(0,0) &= 7r + 4i, \end{aligned}$$

the precise multiplications (5) yield

$$\begin{aligned} (a(0,0))(b(0,0)) &= c(1,3) = 10r + 15i & (8) \\ (a(0,0))(b(1,0)) &= d(1,0) = 18r - i, \end{aligned}$$

and rule (6) directly yields

$$\begin{aligned} c(0,0) &= 15r + 10i & (7'') \\ d(0,0) &= 18r + i. \end{aligned}$$

And we observe that in this specific illustrative case, there indeed does exist an  $e$ :

$$m=17 \quad n=6 \quad \text{whence } e(0,0) = 17r + 6i \quad \text{and } m(e) = 325. \quad (7e)$$

We are now equipped to consider complex integer multiplication in general, while restricting our attention to the primary sector of the complex plane.

We turn to the detailed structure of the sieve-search procedure of the test program.

For integer parameters  $(JN)$  and  $(IN)$  which are greater than 1, with  $(JN)$  at least as large as  $(IN)$ , we define the region of interest (primary sector) as that containing the complex integers  $f(0,0) = gr + hi$  for values of  $g$  up to  $(JN)$  and values of  $h$  up to  $(IN)$ .

We denote the maximum permissible  $\text{modsq}$  over the region of interest as  $M$ ; clearly:  $M = (JN)^2 + (IN)^2$ .

We reject, as irrelevant, all complex integers whose  $\text{modsq}$  exceeds  $M$ .

In Figure 3 we expand Figure 2 to show additional data:  $(IN)$ ,  $(JN)$ , and  $M$  (or, rather, the square root of  $M$ ). (For the specific assignments of (7) (and (7e)), there apply here:  $(IN)=12$ ,  $(JN)=16$ , and  $M=400$ .)

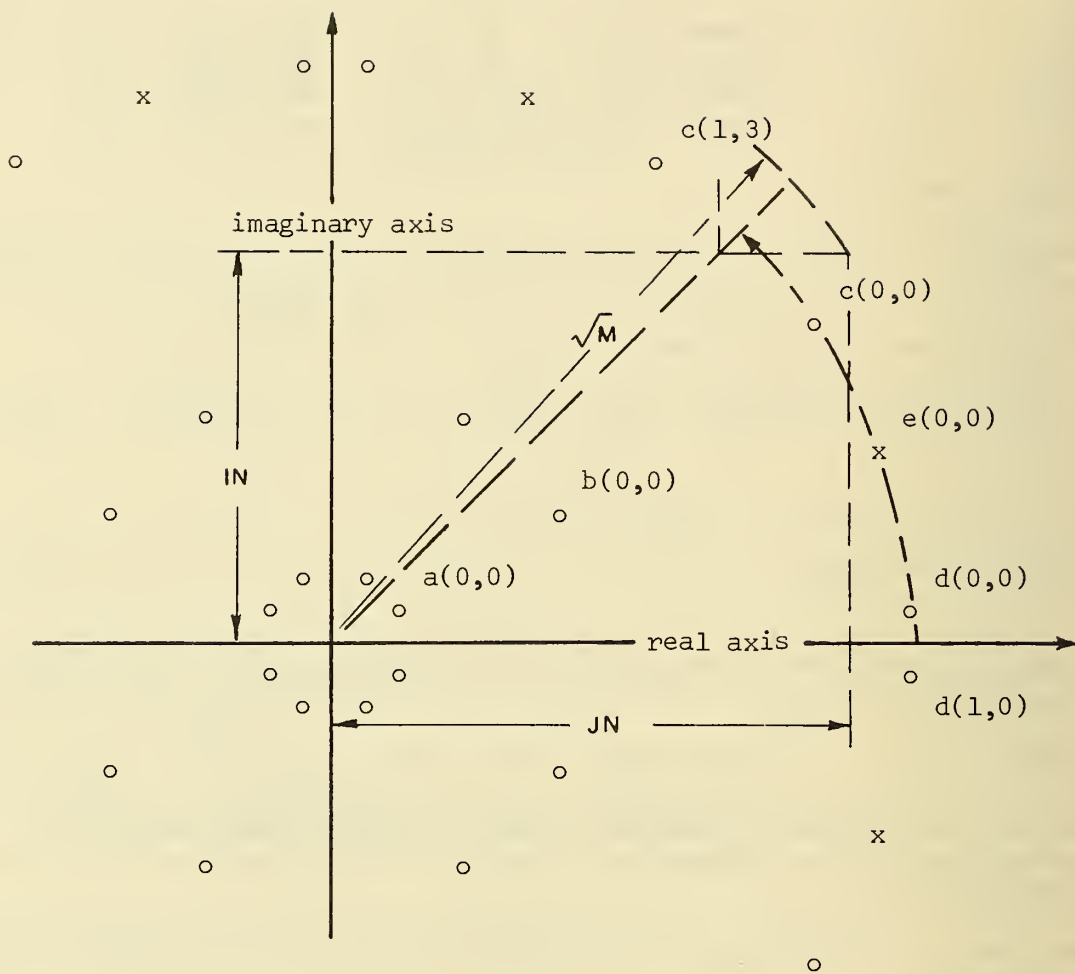


Figure 3.



To discuss our sieve-search procedure, we consider the primary derivants of the two products of the two pairs (k=1,2) of complex integers:

$$\begin{aligned} a(0,0) &= pr + qi & (9) \\ b(k,0) &= sr + \emptyset ti \end{aligned}$$

where

$$\emptyset = (-1)^k. \quad (10)$$

We have the obvious constraints:

$$(m(pr+qi))^2 = (p^2+q^2)^2 \quad \text{less than or equal to } M \quad (11a)$$

$$(m(pr+0i))^2 = (p^2)^2 \quad \text{less than or equal to } M \quad (11p)$$

$$(m(pr+qi))(m(sr+\emptyset ti)) = (p^2+q^2)(s^2+t^2) \quad \text{less than or equal to } M \quad (11b)$$

$$(m(pr+qi))(m(sr+ 0i)) = (p^2+q^2)(s^2) \quad \text{less than or equal to } M, \quad (11s)$$

where (11p) and (11s) are included in (11a) and (11b), respectively.

We force these products to include all composite complex integers in the region of interest. We do so by: (1) requiring that a(0,0) assume, in turn, the succession of values:

1.  $a(0,0) = pr+qi = r + i$  (12a)
2.  $a(0,0) = pr+qi = 2r$
3.  $a(0,0) = pr+qi = 2r + i$
4.  $a(0,0) = pr+qi = 2r + 2i$
5.  $a(0,0) = pr+qi = 3r$
6.  $a(0,0) = pr+qi = 3r + i$
7.  $a(0,0) = pr+qi = 3r + 2i$
- etc.      etc.      etc.,

subject to (11a) until (11p) is violated; and (2) for each a(0,0), requiring that b(k,0) assume, in turn, the succession of values:

1.  $b(k,0) = sr+\emptyset ti = pr + \emptyset qi$  (12b)
2.  $b(k,0) = sr+\emptyset ti = pr + \emptyset(q+1)i$
3.  $b(k,0) = sr+\emptyset ti = pr + \emptyset(q+2)i$
- etc.      etc.      etc.
- n-q+1.  $b(k,0) = sr+\emptyset ti = pr + \emptyset ni$  (n = min (p,(IN)))
- n-q+2.  $b(k,0) = sr+\emptyset ti = (p+1)r$
- n-q+3.  $b(k,0) = sr+\emptyset ti = (p+1)r + \emptyset i$
- n-q+4.  $b(k,0) = sr+\emptyset ti = (p+1)r + 2\emptyset i$
- etc.      etc.      etc.,

subject to (11b) until (11s) is violated.

(A straightforward procedure for merely isolating complex primes would be that of: excluding from (12a) and (12b) all previously recognized composite complex integers; and forming, for each residual pairs ( $a(0,0)$  and  $b(k,0)$ ), the associated primary derivants of the products under (6), ignoring any products which lie outside the region of interest, and marking as composite those which do not. We choose a more complicated (and redundant) procedure, for two reasons: for the especial purpose of employing an innermost loop which has the particular index-employing features we wish to engage, and, incidentally, to elevate representative solution time to reasonably recognizable magnitude.)

The test program employs its innermost programming loop (typified in Table I) immediately subordinate to an obvious four-echelon loop execution of (12a) and (12b), precisely and fully as detailed above.

For each of the pairs  $a(0,0)$  and  $b(k,0)$  which are developed by (12a) and (12b), the innermost loop forms the product  $(m(a))(m(b))$  and searches the region of interest to discover complex integers whose  $\text{modsq}$  are equal to that product. Except in occasional degenerate cases, there exist, perforce, at least two such complex integers: the  $c(0,0)$  and  $d(0,0)$  of (6); but one or both of these may lie outside the region of interest. And there may exist more, as illustrated in (7e).

This search is made generally along the outside edge of the arc shown in Figure 3 --- more precisely: it is made, in the direction shown by the arrow-head, along the outer edge of such portion of that arc as is contained in the region of interest. Indeed, the innermost loop acquires the particular features to which we address our argument by the combined circumstances of: (1) traversing along the outer edge of the arc and (2) accessing the stored table of  $\text{modsq}$ .

For each complex integer for which this equality is discovered, the inner loop: (1) ascertains whether or not it is indeed one of the primary derivants  $c(0,0)$  or  $d(0,0)$  of (6); (2) ignores it if it is not; and (3) raises the pertinent count if it is.

Appendix A Programming

```

COMPLEX SIEVE 6 OCTOBER 66 PAGE 1
CARD SIGNAL TO BEGIN NEW PAGE IN LISTING COMPLEX SIEVE 6 OCTOBER 66, 1
C SIEVE PROGRAM FOR SEPARATING COMPLEX COMPOSITE INTEGERS AND PRIMES 2
C THE * MARKS THE SEVERAL POINTS AT WHICH THIS PROGRAM MUST BE UPDATED * 3
C* 0/ 39 40 999 1 0 4
C ABOVE IS A SAMPLE OF THE INPUT DATA CARD 5
C THE FIRST COLUMN IS IRRELEVANT ON THE DATA CARD 6
C COLUMNS 2-4 CONTAIN OPTIONAL CHARACTERS FOR TABLE PRINTING 7
C THE FIFTH COLUMN IS IRRELEVANT ON THE DATA CARD 8
C COLUMNS 6-30 CONTAIN FIVE FIVE-DIGIT INTEGER PARAMETERS 9
C THE ROLES OF THE FIRST THREE PARAMETERS ARE ACCOUNTED BELOW 10
C THE FIFTH PARAMETER GOVERNS THE METHOD USED FOR TIMING 11
C IF ITIME=+1 THE PROGRAM WILL PAUSE TWICE TO PERMIT MANUAL TIMING 12
C IF ITIME= 0 THE PROGRAM WILL EMPLOY AUTOMATIC CLOCK INTERROGATION 13
C AND TWO APPROPRIATE INTERROGATION STATEMENTS MUST BE EMPLOYED AND 14
C ELAPSED TIME MUST BE EXPRESSED IN SECONDS THROUGH SUBSTITUTING AN 15
C APPROPRIATE ASSIGNMENT TO THE QUANTITY FACTOR WHERE SHOWN BELOW 16
C IF ITIME=-1 BOTH MANUAL AND AUTOMATIC TIMING APPLY 17
C THE FOURTH PARAMETER GOVERNS THE PRINTOUT 18
C IF ILABEL = 1, THE CHARACTER IN COLUMN 2 MARKS PRIMES 19
C IF ILABEL = 1, THE BLANK ( ) IN COLUMN 3 MARKS COMPOSITE NUMBERS 20
C IF ILABEL = 1, THE CHARACTER IN COLUMN 4 MARKS THE UNUSED CORNER 21
C IF ILABEL = 0, THE DIGIT 0 MARKS COMPOSITE NUMBERS 22
C IF ILABEL = 0, THE DIGIT 1 MARKS PRIMES 23
C 24
C EMPLOY TABLE M(-,-) OF SQUARES OF MODULI AND INITIAL PRIME MARKERS 25
C 26
C E 1 1 1 1 1 - - ...JN=7 MARKERS ARE - 27
C X 1 1 1 1 1 - - 6 ABOVE THE - 28
C A F JNO=6 1 1 1 1 32 41 52 ...IN=5 DIAGONAL * 29
C M O AND 1 1 1 18 25 36 45 4 * * 30
C P R INO=4 1 1 8 13 20 29 40 3 * * SQUARES 31
C L 1 2 5 10 17 26 37 2 * * ARE BELOW 32
C E 0 1 4 9 16 25 36 1 * * THE DIAGONAL 33
C 34
C THE PROGRAMMING SEARCHES TABLE M(-,-) FOR COMPOSITE NUMBERS UNDER 35
C A PARTICULAR SCHEME AND COUNTS THE NUMBER OF RECOGNITIONS OF EACH 36
C UNDER THE SEARCH SCHEME EVERY POSSIBLE PRODUCT IS FORMED OF 37
C COMPLEX NUMBERS WITHIN THE RANGE OF INTEREST IN THE COMPLEX PLANE 38
C 39
C DIMENSION MARKS(5) 40
C LJP MUST BE AT LEAST 5 LESS THAN THE NUMBER OF PRINTING POSITIONS 41
C LJP MUST BE AT LEAST 100 AND MUST NOT EXCEED 195 42
C LJN MUST NOT EXCEED LJP 43
C* DIMENSION LABLE(LJP) 44
C DIMENSION LABLE(115) 45
C* DIMENSION M(LJN,LJN) (UPPER CORNER (ABOVE IN,IN) WILL NOT BE USED) 46
C DIMENSION M( ) 47
C* LJP= 48
LJP=115 49
C* LJN= 50
LJN= 51
C* THE ABOVE ELEVEN LINES MUST BE COMPATABLE 52
C CHECK FOR LJN .LE. LJP AND 100 .LE. LJP .LE. 195 53
IF(LJN=LJP) 40, 40, 999 54
40 IF(100=LJP) 41, 41, 999 55
41 IF(LJP=195) 42, 42, 999 56
42 CONTINUE 57

```

```

CARD SIGNAL TO BEGIN NEW PAGE IN LISTING
C IDENTIFY I/O UNITS .... IDI FOR INPUT AND IDO FOR OUTPUT
C* IDI=
IDI=
C* IDO=
IDO=
C FORMATS
100 FORMAT(1H )
101 FORMAT(1H1)
102 FORMAT(9I6,18,F10,3)
103 FORMAT( 5A1,5I5)
106 FORMAT(54H1 MIN MAX DIAG DIAG COMP TOTAL INDEX INDEX TALLY
1 18H KEY TIME IN)
107 FORMAT(54H COEFF COEFF LIMIT COUNT COUNT IMAX JMAX MAX
1 18H COUNT SECONDS)
C*110 FORMAT(1H 13,1H (LJP)I1)
110 FORMAT(1H 13,1H 115 I1)
C*111 FORMAT(1H 13,1H (LJP)A1)
111 FORMAT(1H 13,1H 115 A1)
C*130 FORMAT( 5X,(LJP )I1)
130 FORMAT( 5X, 115 I1)
C*131 FORMAT( 15X,(LJP- 10)I1)
131 FORMAT( 15X, 105 I1)
C*132 FORMAT(105X,(LJP-100)I1)
132 FORMAT(105X, 15 I1)
C* THE ABOVE TEN LINES MUST BE COMPATABLE
C READ AND WRITE AND CHECK PARAMETERS (2,LE,INO,LE,JNO,LT,LNJ)
C INO,JNO ARE LIMITS ON VALUES OF COEFFICIENTS OF COMPLEX NUMBERS
C IN=INO+1 AND JN=JNO+1 ARE CORRESPONDING TABLE INDEX LIMITS
C KN IS MAXIMUM ADMISSIBLE NUMBER OF PRODUCTS ON PRINCIPAL DIAGONAL
C THE PROGRAMMING ABORTS UPON THE (KN)TH RECOGNITION OF A COMPLEX
C NUMBER ON THE DIAGONAL RENDERING PRIME IDENTIFICATION INCONCLUSIVE
C EXCEPT KN=0 PERMITS UNLIMITED PRODUCTS ON THE PRINCIPAL DIAGONAL
C READ (IDI,103) (MARKS(I),I=1,5), INO,JNO,KN,ILABLE,ITIME
WRITE (IDO,101)
WRITE (IDO,102) INO,JNO,KN,ILABLE,ITIME
IF(2-INO) 162, 162, 999
162 IF(INO-JNO) 163, 163, 999
163 IF(JNO-LJN) 164, 999, 999
164 IN=INO+1
JN=JNO+1
C COMPUTE SQUARES OF MODULI AND SET PRIME MARKERS AS IN TABLE ABOVE
DO 167 J=1,LJN
167 M(1,J)=(J-1)**2
DO 169 I=2,LJN
DO 169 J=1,LJN
M(J,I-1)=1
169 M(I,J)=M(1,I)+M(1,J)
C INITIALIZE COUNTERS FOR DIAGONAL PRODUCTS, COMPOSITE NUMBERS, KEY
KO=0
KC=0
KEY=0

```



```

CARD SIGNAL TO BEGIN NEW PAGE IN LISTING
C   PRESET TIMING READINGS TO ZEROS
    TIMEON=0.
    TIMEUP=0.
C   INTERROGATE CLOCK
    IF(ITIME)
190 PAUSE
    IF(ITIME)
191 CONTINUE
C   INSERT CLOCK INTERROGATION HERE TO DETERMINE TIMEON
C*  CALL CLOCK FUNCTION FOR TIMEON
    CALL
    TIMEON=
199 CONTINUE
C   ESTABLISH COMPOSITE NUMBERS AS PRODUCTS OF LOWER AND UPPER FACTORS
C   RANGE LOWER FACTOR, EXCLUDING UNIT PRIME AND EXCESSIVE MODULUS
    IE=2
C   IE=1 WILL BE SET AS FINAL ACTION OF EACH ITERATION OF LOOP JL=2,JN
    DO 298 JL=2,JN
    IF(M(1,JL)**2-M(IN,JN))
207 DO 297 IL=IE,IN
    IF(IL-JL)
C   RANGE UPPER FACTOR, EXCLUDING EXCESSIVE PRODUCT MODULUS
212 DO 296 JU=JL,JN
    IF(M(IL,JL)*M(1,JU)-M(IN,JN))
217 DO 295 IU=1,IN
    IF(IU-JU)
C   EXCLUDE IU LESS THAN IL WHEN JL EQUALS JU
222 IF(JL-JU)
223 IF(IL-IU)
C   COMPUTE RADIUS MODULUS (SQUARED)
227 MODRAD=M(IL,JL)*M(IU,JU)
C   EXCLUDE EXCESSIVE PRODUCT MODULUS
    IF(MODRAD-M(IN,JN))
C   LOCATE INITIAL INDICIES FOR SEARCH FOR PRODUCT MODULUS
232 IO=1
    J=JN
    IF(MODRAD-M(1,JN))
C   INITIATE SEARCH FROM REAL AXIS AT (1,J)
242 DO 244 J=2,JN
    IF(MODRAD-M(1,J))
244 CONTINUE
C   INITIATE SEARCH FROM REAL=(JU-1) AT (IO,JN)
247 DO 249 IO=1,IN
    IF(MODRAD-M(IO,J))
249 CONTINUE
C   COMPUTE (INDICES OF) REAL COORDINATES OF PRODUCTS
252 JX=(JL-1)*(JU-1)-(IL-1)*(IU-1)+1
    JR=(JL-1)*(IU-1)+(JU-1)*(IL-1)+1
    IF(JX-JR)
255 JX=JR
256 JR=(JL-1)*(JU-1)+(IL-1)*(IU-1)+1

```



CARD SIGNAL TO BEGIN NEW PAGE IN LISTING				162
C LOCATE PRODUCT MODULI (EXCEPTING THOSE OUTSIDE REGION OF CONCERN)				163
DO 279 I=10,IN				164
C BRACKET MODULUS WITH INDEX J				165
IF(I-J)	261,	261,	295	166
260 J=J-1				167
KEY=KEY+1				168
261 IF(MODRAD-M(I,J))	262,	266,	999	169
262 IF(I-J)	263,	295,	295	170
263 IF(MODRAD-M(I,J-1))	260,	260,	279	171
C CHECK PRODUCT COORDINATE AGREEMENT FOR FIRST RECOGNITION				172
266 IF(J-JR)	273,	269,	279	173
C SEPARATE DIAGONAL CASE FOR FIRST RECOGNITION				174
269 IF(I-J)	270,	287,	295	175
C TALLY NON-DIAGONAL CASE				176
270 M(J,I)=M(J,I)+1				177
C EXCLUDE DUPLICATE CASES FROM SECOND RECOGNITION CHECK				178
IF(JX-JR)	279,	295,	295	179
C CHECK PRODUCT COORDINATE AGREEMENT FOR SECOND RECOGNITION				180
273 IF(J-JX)	295,	280,	279	181
C CONTINUE SEARCH				182
279 CONTINUE				183
C SECOND NON-DIAGONAL PRODUCT MODULUS NOT LOCATED WITHIN RANGE OF IN				184
GO TO 295				185
C SEPARATE DIAGONAL CASE FOR SECOND RECOGNITION				186
280 IF(I-J)	283,	287,	295	187
C TALLY NON-DIAGONAL CASE				188
283 M(J,I)=M(J,I)+1				189
C TERMINATE SEARCH ON SECOND (NON-DIAGONAL) TALLY				190
GO TO 295				191
C TALLY AND COUNT DIAGONAL CASE				192
287 IF(KN)	999,	295,	288	193
288 KO=KO+1				194
IF(KO-KN)	295,	299,	999	195
C LOOP ENDINGS				196
295 CONTINUE				197
296 CONTINUE				198
297 CONTINUE				199
298 IE=1				200
C END OF REITERATION				201
299 CONTINUE				202

```

CARD SIGNAL TO BEGIN NEW PAGE IN LISTING                203
C   INTERROGATE CLOCK                                    204
    IF(ITIME)                                           300, 300, 301  205
300 CONTINUE                                           206
C   INSERT CLOCK INTERROGATION HERE TO DETERMINE TIMEUP  207
C*  CALL CLOCK FUNCTION FOR TIMEUP                     208
    CALL                                               209
    TIMEUP=                                           210
    IF(ITIME)                                           301, 302, 302  211
301 PAUSE                                             212
302 CONTINUE                                           213
C   COMPUTE ELAPSED TIME IN SECONDS BY APPROPRIATELY ASSIGNED FACTOR  214
    FACTOR=1.                                           215
C*  FACTOR=                                           216
    FACTOR=                                           217
    TIME=(TIMEUP*TIMEON)*FACTOR                       218
C   FINAL SETTING OF DIAGONAL AND FILL-OUT OF POSITIVE QUARTER-PLANE  219
C   AND SEARCH FOR MAXIMUM NON-DIAGONAL COMPOSITE NUMBER TALLY      220
    MAXC=0                                             221
    DO 306 I=1,IN                                     222
C   SET COMPOSITE DIAGONAL MARKS (ADJUST M(2,2) LATER)  223
    M(I,I)=0                                           224
    DO 306 J=I,JN                                     225
    IF(MAXC-M(J,I))                                   303, 304, 304  226
C   RECORD LARGEST COUNT DATA                          227
303 MAXC=M(J,I)-1                                     228
    MAXJ=J-1                                           229
    MAXI=I-1                                           230
304 IF(M(J,I)-1)                                     306, 306, 305  231
305 KC=KC+M(J,I)-1                                   232
    M(J,I)=0                                           233
306 M(I,J)=M(J,I)                                     234
C   FINAL ADJUSTMENT FOR LONE DIAGONAL PRIME M(2,2)=1  235
    M(2,2)=1                                           236
C   COMPUTE TOTAL TALLY                                 237
    KT=KC+KO                                           238

```

```

CARD SIGNAL TO BEGIN NEW PAGE IN LISTING
C OUTPUT
C WRITE KEY DATA
WRITE (IDO,106)
WRITE (IDO,107)
WRITE (IDO,102) INO,JNO,KN,KO,KC,KT,MAXI,MAXJ,MAXC,KEY,TIME
WRITE (IDO,100)
C WRITE POSITIVE QUARTER PLANE
DO 320 I=1,JN
NN=JN
IF(I-(JN-IN))
313 NN=IN
314 K=JN-(I-1)
DO 315 L=1,JN
315 LABEL(L)=MARKS(4)
DO 316 L=1,NN
KK=3-M(K,L)
316 LABEL(L)=MARKS(KK)
L=K-1
IF(ILABEL)
C CHECK DISTINCTION BETWEEN CHARACTERS IN DATA CARD COLUMNS 2 AND 3
317 IF(MARKS(2)-MARKS(3))
318 WRITE (IDO,111) L,(LABEL(KK),KK=1,JN)
GO TO 320
319 WRITE (IDO,110) L,(M(K,KK),KK=1,NN)
320 CONTINUE
C WRITE INDICES
DO 322 I=1,10
322 LABEL(I)=I-1
C HUNDREDS LINE
IF(JN-100)
324 WRITE (IDO,100)
GO TO 332
327 DO 328 I=101,JN
328 LABEL(I)=LABEL(2)
WRITE (IDO,132) (LABEL(I),I=101,JN)
C TENS LINE
332 IF(JN-10)
334 WRITE (IDO,100)
GO TO 347
337 DO 340 I=11,JN
J=(I-1)/10+1
IF(J-10)
339 J=J-10
340 LABEL(I)=LABEL(J)
WRITE (IDO,131) (LABEL(I),I= 11,JN)
C UNITS LINE
DO 344 I=11,JN
344 LABEL(I)=LABEL(I-10)
347 WRITE (IDO,130) (LABEL(I),I= 1,JN)
WRITE (IDO,101)
C TERMINAL
999 STOP
END

```

Appendix B Sample Output

MIN	MAX	DIAG	DIAG	COMP	TOTAL	INDEX	INDEX	TALLY	KEY	TIME IN
COEFF	CJEFF	LIMIT	COUNT	COUNT	COUNT	IMAX	JMAX	MAX	COUNT	SECONDS
39	40	999	138	1839	1977	20	40	14	8636	0

```

40 * * * * *
39 * * * * *
38 * * * * *
37 * * * * *
36 * * * * *
35 * * * * *
34 * * * * *
33 * * * * *
32 * * * * *
31 * * * * *
30 * * * * *
29 * * * * *
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9 * * * * *
8 * * * * *
7 * * * * *
6 * * * * *
5 * * * * *
4 * * * * *
3 * * * * *
2 * * * * *
1 * * * * *
0 * * * * *

```

1111111111222222222233333333334  
01234567890123456789012345678901234567890





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